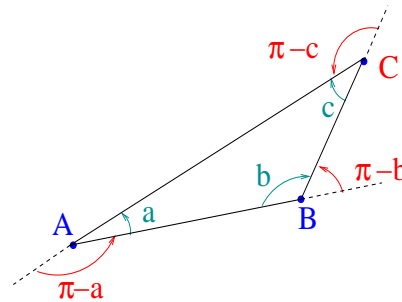


THE SUM OF THE INTERIOR ANGLES OF A TRIANGLE

The following figure provides the basic geometric explanation of the fact that the sum of the interior angles of any triangle has to be equal to π radians (or 180 degrees)

FIG. 1:

The sum of the interior angles of a triangle is π radians:



Start at A and go once counterclockwise around the triangle (hence $+2\pi$ radians overall) or, in steps, first counterclockwise by $\pi-b$ at B, then by $\pi-c$ at C and finally by $\pi-a$ at A, for a total of $3\pi-a-b-c$ in all
Equating $3\pi-a-b-c=2\pi$ then yields $a+b+c=\pi$ as claimed

- Note that, in the figure, use has been made of the fact that, at each of the vertices A, B and C of the triangle, the interior and exterior angles at the vertex combine to form a half of a full revolution, implying that their sum must be π radians (180 degrees). Since the interior angle at A is denoted a , this implies the exterior angle at A must be $\pi - a$ etc.

- If we specialize to right angle triangles, one of the angles is $\pi/2$ radians, and hence if the two other angles are called θ and ϕ , we must have $\frac{\pi}{2} + \theta + \phi = \pi \Rightarrow \phi = \pi - \frac{\pi}{2} - \theta = \frac{\pi}{2} - \theta$. Similarly, $\theta = \frac{\pi}{2} - \phi$.
- The discussion in the last point shows that, in a right angle triangle, once one is given the value of one of the two non-right angles in the triangle, the other such angle, and hence all three of the angles in the triangle are specified.
- *Another way of phrasing the last point is to say that, in dealing with right angle triangles, the value of one of the non-right angles completely specifies the SHAPE (though not the overall size) of the triangle.*

EXERCISES:

(In the exercises that follow, all angles are given in radians, and all angles occurring in your answers should also be given in radians)

1. If one of the non-right angles in a right-angle triangle is $\pi/3$, determine the other non-right angle in the triangle.
2. If one of the non-right angles in a right-angle triangle is $\pi/4$, determine the other non-right angle in the triangle.
3. If one of the non-right angles in a right-angle triangle is $2\pi/7$, determine the other non-right angle in the triangle.
4. A triangle has one internal angle equal to $2\pi/5$ and another internal angle equal to $\pi/10$. Determine whether or not the triangle is a right-angle triangle.
5. A triangle has one internal angle equal to $3\pi/11$ and another internal angle equal to $2\pi/11$. Determine whether or not the triangle is a right-angle triangle.
6. Is it possible to find a triangle with internal angles $3\pi/14$, $5\pi/14$ and $3\pi/7$?
7. Is it possible to find a triangle with internal angles $13\pi/35$, $\pi/7$ and $3\pi/5$?