THE UNIT CIRCLE PICTURE FOR FIRST QUADRANT DIRECTIONS

Recall the basic geometric definitions of the six trig functions, from Chapter 2 of the course: In a right-angle triangle with one non-right angle θ, if h is the length of the hypoteneuse of a right angle triangle, a is the length of the side adjacent to θ, and o is the length of the side opposite to θ, then

$$\begin{aligned} sin(\theta) &= o/h, & cos(\theta) = a/h \\ tan(\theta) &= o/a, & cot(\theta) = a/o \\ csc(\theta) &= h/o, & sec(\theta) = h/a \end{aligned}$$

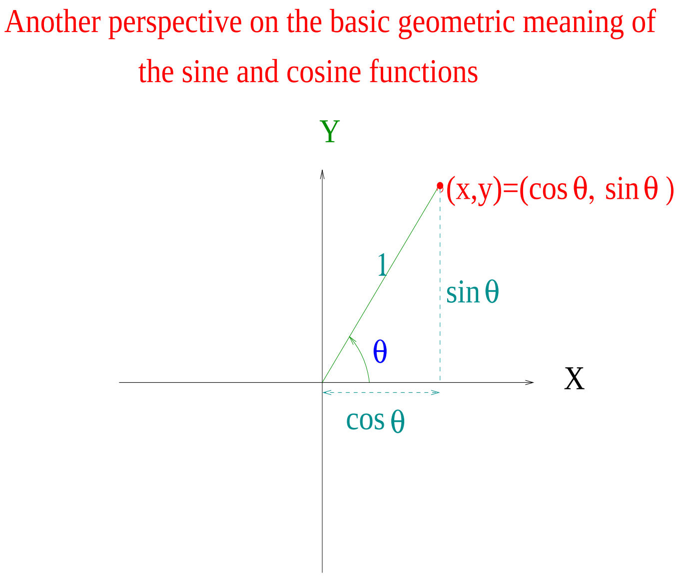
From these definitions, it follows, using the same notation, that

$$\begin{aligned} a &= h\,cos(\theta) \\ o &= h\,sin(\theta) \\ o &= a\,tan(\theta) \\ a &= o\,cot(\theta) \\ h &= o\,csc(\theta) \\ h &= a\,sec(\theta) \end{aligned} \quad (1)$$

This apparently innocuous rearrangement turns out to be very useful, for the reasons discussed below, connected to Figure 1.

- The first two of Equations 1, specialized to the triangle of the given shape whose size is such that the length of the hypoteneuse is 1, gives us a nice, easily visualized, geometric characterization of the sine and cosine, as indicated in Fig. 1.
- Note that, from the figure, the x coordinate of the tip of the hypoteneuse is $cos(\theta)$, and the y coordinate of the tip of the hypoteneuse is $sin(\theta)$.

FIG. 1:



We can thus interpret $sin(\theta)$ and $cos(\theta)$ in either of the following ways:

- $sin(\theta)$ is the length of the opposite side of the right-triangle with shape characterized by angle θ and hypoteneuse 1, while $cos(\theta)$ is the length of the adjacent side of this same triangle, or
- $(\,cos(\theta),\, sin(\theta)\,)$ are the (x, y) coordinates of the tip of a line of length 1 oriented at an angle θ to the positive x axis.

- This way of characterizing the sine and cosine functions is sometimes called "the unit circle picture".

FIG. 2:

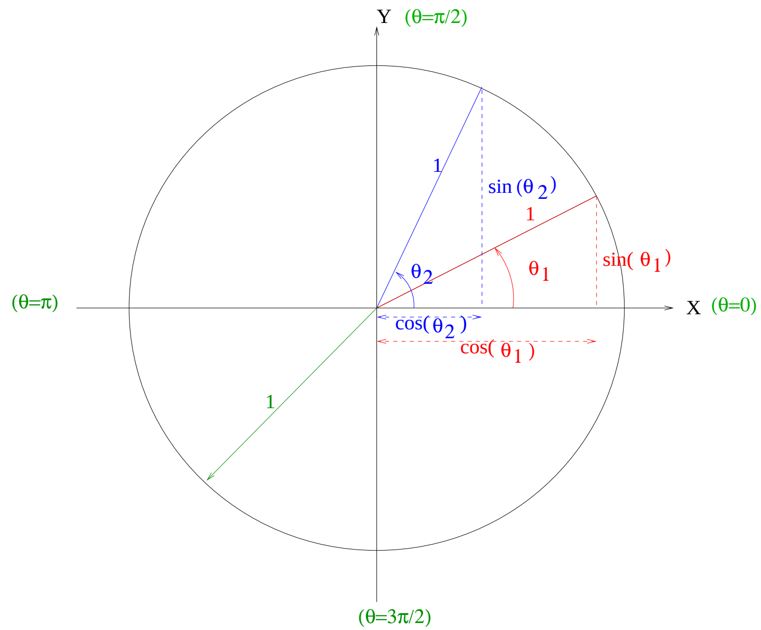


Figure 2 illustrates the first of the two perspectives above for two different angles, and indicates why this is a sensible terminology.

- Figure 2 also illustrates the utility of the unit circle perspective for finding out how $sin(\theta)$ and $cos(\theta)$ as the angle θ is changed: it becomes geometrically obvious that, for two angles like those shown in the figure, with $0 < \theta_1 < \theta_2 < \pi/2$,

$$cos(\theta_2) < cos(\theta_1) \ \ and \ \ sin(\theta_2) > sin(\theta_1) \, .$$

- One does not have to memorize how the sine and cosine functions change as the angle increases in the first quadrant, one can just look at the pictures and use basic geometrical intuition.

- The second of the characterizations/visualizations above (involving the interpretation of the cosine and sine as x and y coordinates of the tip of a hypoteneuse of length *1*) turns out to be especially useful when one goes to generalize the trig functions to angles which no longer lie in the first quadrant, a topic which will be discussed in the next section of Chapter 3.

EXERCISES:

In the exercises that follow, all angles are given in radians. Use the relations in Equations 1, together with what you know about the trig functions of the special first quadrant angles $\pi/6$*,* $\pi/4$ *and* $\pi/3$ *to solve these problems. If needed, you can review the trig functions of these spectial angles by returning to the relevant section of Chapter 2 of the course.*

1. If one of the non-right angles of a right-angle triangle is $\theta = \pi/6$ and the hypoteneuse has length *6*, find the lengths of the sides adjacent and opposite to θ.
2. If one of the non-right angles of a right-angle triangle is $\theta = \pi/4$ and the hypoteneuse has length $2\sqrt{2}$, find the lengths of the sides adjacent and opposite to θ.
3. If one of the non-right angles of a right-angle triangle is $\theta = \pi/3$ and the length of the side adjacent to θ is *4*, find the length of the hypoteneuse and the length of the side opposite to θ.
4. If one of the non-right angles of a right-angle triangle is $\theta = \pi/3$ and the length of the side opposite to θ is *3/2*, find the length of the hypoteneuse and the length of the side adjacent to θ.
5. If one of the non-right angles of a right-angle triangle is $\theta = \pi/4$ and the length of the side opposite to θ is $\sqrt{2}$, find the length of the hypoteneuse and the length of the side adjacent to θ.